# Inviscid Solution of the Steady, Hypersonic Near Wake by a Time-Dependent Method

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**The steady near wake of a slender, two-dimensional flat-based body at hypersonic velocity is analyzed within the context of an inviscid, rotational flow. The upstream boundary layer is considered the source of vorticity convected into the base region; the generation of additional viscous stresses within this region, heat transfer to the base, and diffusion of the vorticity are neglected. The over-all structure of the base flow pattern is the result of an inviscid interaction between the subsonic, recirculating flow and the outer, supersonic rotational stream. In this sense the proposed model represents the first term in the conventional asymptotic expansion in inverse powers of Reynolds number by which viscous effects are identified and systematically determined. Numerical solutions are carried out by a time-dependent method known as the two-step Lax-Wendroff technique. The steady-state solution is obtained asymptotically; in this manner the complexities associated with the steady problem are effectively circumvented. Comparisons with experimental data are presented.**

## Nomenclature

$C_p,C_v$ = specific heats at constant pressure and constant volume
$E$ = total energy, also boundary value of $e$
$e$ = vector containing the four dependent variables, $\rho$, $u$, $v$, and $T$
$f,g$ = vector functions of $e$, defined by Eq. (1)
$H$ = total enthalpy, also base height
$L$ = reference length scale
$M$ = Mach number
$p$ = pressure
$q$ = velocity vector
$R$ = gas constant
$Re,Re_H$ = Reynolds number, Reynolds number referred to the base height
$T$ = temperature
$t$ = time
$u,v$ = components of velocity in the $x$ and $y$ directions, respectively
$x,y$ = Cartesian coordinates aligned with, and normal to, the freestream velocity vector
$\delta$ = boundary-layer thickness
$\Delta$ = incremental operator, also mesh size
$\Phi$ = heuristic factor defined by Eq. (4)
$\rho$ = density
$\psi$ = streamfunction

### *Superscripts*

$(p)$ = provisional value
$\bar{\ }$ = average value
$\hat{\ }$ = dimensional variable

### *Subscripts*

$b$ = base value, specifically at the point $x = 0, y = 0$
$e$ = edge of boundary layer
$w$ = wall value
$0$ = total or stagnation value
$x,y,t$ = differentiation with respect to $x$, $y$, or $t$
max = value at outer boundary of domain
min = minimum value
$\infty$ = freestream value
$\mathbb{C}$ = centerline, or plane of symmetry

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Index Categories: Jets, Wakes, and Viscid-Inviscid Flow Interactions; Supersonic and Hypersonic Flow.

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## I. Introduction

NUMEROUS aerodynamic phenomena of current interest involve mixed subsonic/supersonic flows; the transonic airfoil and supersonic blunt body problems are several familiar examples. Another example, not usually considered in the context of an inviscid flow, is the near wake of a bluff-based body at supersonic velocity. The present investigation is directed toward advancing the understanding of the latter problem as well as the development of a powerful numerical method for analysis of this general category of mixed flows.

The flow in the immediate vicinity of the base of a slender wedge at supersonic velocity is treated within the framework of an inviscid interaction between the supersonic, rotational flow, which originates in the upstream boundary layer, and the adjacent subsonic vortices held at the base. The problem can be considered analogous in certain respects to the supersonic blunt body problem; the elliptic character of the subsonic region must be recognized in both problems, and the solutions must account for the related upstream propagation of signals. On the other hand, in the blunt body problem the effects of viscosity are usually confined to the shock wave, which can be treated as a surface of discontinuity at even moderately low Reynold's number. In the near-wake problem the dividing streamline (which separates the recirculating flow from that which continues downstream) can be correctly assumed to be a contact surface only in the limit of infinite Reynolds number. Determination of the roles of diffusion, dissipation, heat conduction, etc., is clearly of practical concern in this regard. However, proper categorization of the problem as either a strong or weak viscous interaction requires statement of the relative magnitudes of inertial and viscous forces. Establishment of a limiting solution for infinite Reynolds number is relevant to identification of the inertial forces. Furthermore, this limiting solution represents the leading term in the asymptotic expansion by which weak viscous interaction effects can be identified and systematically determined.[1]

The near-wake model proposed herein derives in part from the inviscid model for incompressible flows proposed by Batchelor,[2] viz., a pair of counter rotating vortices bound to the base by a vortex sheet, and in part from the inviscid boundary-layer expansion models of Zakkay and Tani[3] and Weiss.[4] It is recognized that the transonic, rotational flow resulting from the passage of the boundary layer over a sharp, expansive corner produces normal and streamwise pressure gradients of comparable magnitude in the immediate vicinity of the corner, which are balanced by purely inertial forces.[5] (Viscous effects are confined to a sublayer having thickness of the order $Re^{-3/4}$.) Furthermore, the assumption that the viscous stresses produced within the boundary layer are effectively quenched by the expansion at hypersonic conditions[4] (an idea first proposed in Ref. 3) has been verified by Ohrenberger[6] using the experimental data reported by Batt and Kubota.[7] The proposed model retains the vorticity convected into the base region from the upstream boundary layer; the additional viscous stresses generated within the base region and the diffusion thereof are not considered. The present analysis and resulting solutions therefore pertain to the near wake formed by separation of a hypersonic boundary layer from a sharp cornered, flat based body. Comparisons are made with data from Refs. 7 and 8. The situation of interest is distinct from that for a blunt body, in which case the boundary-layer identity may be retained through the separation process, and the Reeves and Lees[9] model is descriptive.

The near-wake problem for steady flow in the limit of vanishing viscosity is of the mixed elliptic/hyperbolic type; however, the problem is rendered purely hyperbolic by seeking the steady-state solution as the asymptote of a time-dependent problem subject to steady boundary conditions. This concept has received considerable attention in connection with highly complex flows.[10–14]

## II. Method of Solution

### A. Background

A finite difference technique generally attributed to Lax[15,16] is employed herein. Several general remarks are considered warranted concerning the mathematical and physical implications of the method prior to discussion of its implementation and the results thereof. Lax[15] pointed out that solutions of hyperbolic equations which contain surfaces of discontinuity (termed weak solutions) are not, in general, unique; uniqueness is a consequence of imposition of some additional constraint not implicit in the governing equations. In the case of flow across shock waves, the well-known additional constraint is provided by the second law of thermodynamics. Correspondingly, numerical solutions of the conservation laws that purport to represent flows with embedded shocks must be consistent with the requirements of the second law of thermodynamics (if it is not explicitly contained in the statement of the problem). The Lax-Wendroff technique[16] obtains the correct constraint in the solution through formulation such that the difference equations identically satisfy the conservation laws to within the accuracy of the difference approximations, while the remaining truncation error produces a dissipative effect, viz., increases the entropy of the flow. Consequently, a property of numerical dissipation or artificial viscosity is attributed to the technique. The solutions obtained through its use are therefore considered inviscid in the sense that the effects of artificial viscosity are confined to layers whose thickness is of the order the mesh size, which is sensibly thin on the scale of the problem at hand, and the existence of a limit can be demonstrated as the mesh size is progressively diminished.

Although a nonsteady solution at early time depends continuously on the initial data, (which is, in point of fact, usually arbitrary) the asymptotic solution is independent of the initial data and is only a function of the boundary conditions. It is, of course, implied that the boundary conditions are steady; however, Moretti[17,18] has pointed out that certain difference approximations imposed at the boundaries may be inconsistent with the correct boundary conditions and can thereby preclude attainment of a steady asymptotic solution. Therefore, particular attention is devoted herein to statement of the difference approximations at the boundaries.

### B. Statement of the Governing Equations

The governing equations can be derived in difference form from first principles, subject to the requirement that the density, velocity and temperature are at least piece-wise continuous, integrable functions. It is somewhat more convenient to accept the well-known differential form of the conservation equations as basic, and use Taylor series expansions to derived the corresponding difference equations, to any desired degree of accuracy. With this in mind, the governing equations are stated in the conservation law form suggested by Lax:

$$e_t + f_x + g_y = 0 \tag{1}$$

where

$$e = \begin{vmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{vmatrix} \qquad f = \begin{vmatrix} \rho u \\ \rho(u^2 + T) \\ \rho uv \\ \rho uH \end{vmatrix} \qquad g = \begin{vmatrix} \rho v \\ \rho uv \\ \rho(v^2 + T) \\ \rho vH \end{vmatrix}$$

$$E = C_v T + (u^2 + v^2)/2 \qquad C_v = \tfrac{5}{2} \qquad v = \hat{v}/q_\infty$$

$$H = C_p T + (u^2 + v^2)/2 \qquad C_p = \tfrac{7}{2} \qquad T = \hat{R}\hat{T}/\hat{q}_\infty{}^2$$

$$\rho = \hat{\rho}/\hat{\rho}_\infty \qquad x = \hat{x}/\hat{L} \qquad t = \hat{t}\hat{q}_\infty/\hat{L}$$

$$u = \hat{u}/\hat{q}_\infty \qquad y = \hat{y}/\hat{L} \qquad \hat{p} = \hat{\rho}\hat{R}\hat{T}$$

The freestream Mach number is $M_\infty = \hat{q}_\infty/(1.4\, \hat{R}\hat{T}_\infty)^{1/2}$; thus $\hat{T}/\hat{T}_\infty = 1.4\, M_\infty{}^2 T$ and $\hat{p}/\hat{p}_\infty = 1.4\, M_\infty{}^2 \rho T$.

### C. Finite-Difference Formulation

Formulation of the difference equations used herein derives from the Richtmyer[10] variation of the Lax-Wendroff[16] technique, termed the "two-step" method. The first step is to obtain provisional values at the midpoint of the time step:

$$e^{(p)}(x,y,t + \Delta t/2) = \bar{e}(x,y,t) - [f_x(x,y,t) + g_y(x,y,t)]\Delta t/2 \tag{2}$$

where:

$$f_x(x,y,t) = [f(x + \Delta x,y,t) - f(x - \Delta x,y,t)]/(2\Delta x)$$

$$g_y(x,y,t) = [g(x,y + \Delta y,t) - g(x,y - \Delta y,t)]/(2\Delta y)$$

$$\bar{e}(x,y,t) = [e(x + \Delta x,y,t) + e(x - \Delta x,y,t) + e(x,y + \Delta y,t) + e(x,y - \Delta y,t)]/4$$

Then the second step gives:

$$e(x,y,t + \Delta t) = e(x,y,t) - [f_x{}^{(p)}(x,y,t + \Delta t/2) + g_y{}^{(p)}(x,y,t + \Delta t/2)]\Delta t + \Phi e_d(x,y,t)/3 \tag{3}$$

where:

$$f_x{}^{(p)} = f_x(e_x{}^{(p)}),\ g_y{}^{(p)} = g_y(e^{(p)}),\ e_d = \bar{e} - e$$

and $\Phi$ is a heuristic parameter in the range $0 < \Phi < 1$, which will be discussed in the next section.

### D. Stability Considerations

The stability limitations which must be imposed on Eqs. (2) and (3) can presumably be estimated by application of standard considerations[10] to the linear equation $e_t + Ae_x + Be_y = 0$ with constant coefficients, if the actual coefficients corresponding to Eq. (1) are reasonably smooth.[16] An obvious caveat must be recognized if the coefficients change very fast or nearly discontinuously; the last term of Eq. (3) has been introduced here to suppress possible oscillations due to such nonlinearities. In the ensuing analysis and computations a square grid is used, i.e., $\Delta x = \Delta y$. Linear stability analysis indicates Eq. (3) should be stable, with $\Phi = 0$, for[13]:

$$\Delta t/\Delta x \leq \min(1/\{8^{1/2}[(1.4T)^{1/2} + |u|]\},\ 1/\{8^{1/2}[(1.4T)^{1/2} + |v|]\}) \quad (4)$$

In the present application of this method, the above stability limit has been interpreted conservatively as:

$$\Delta t/\Delta x \leq M_\infty/3(1 + 5^{1/2})(1 + M_\infty{}^2/5)^{1/2} \quad (5)$$

Nonetheless, the solution of Eq. (3) subject to the equality expressed in Eq. (5) proved to be only neutrally stable with $\Phi = 0$ for the cases of present interest, i.e., oscillations that developed during the transient phase did not necessarily damp out completely in time, although they did remain bounded. This behavior of the $\Phi = 0$ solution is conjectured to be a failure of the method to damp certain high-frequency (nonlinear) harmonics associated with the highly rotational flows considered herein. Numerical experimentation revealed that values of $0 < \Phi \leq \Delta x$ provide the required stability, while still retaining second order accurcy. The dependence of the solution on $\Phi$, and the existence of a limit as $\Phi \to 0$, is demonstrated a posteriori in Ref. (19).

### E. Boundary Conditions

The boundary conditions prescribed on the problem under consideration are formulated with reference to the general configuration sketched in Fig. 1. The locations of the outer boundaries are arbitrary, as will be discussed, and they were varied in the range $\frac{3}{4} \leq y/H \leq \frac{3}{2}$ and $1 \leq x/H \leq 2$ during the course of the study.

The dependent variables on the inflow boundary $x = 0$ and $y \geq H/2$ are specified functions e.g.:

$$e(x,y,t) = E(y) \qquad \text{on} \begin{cases} x = 0 \\ H/2 \leq y \leq 3H/2 \\ t \geq 0 \end{cases} \quad (6)$$

The determination of $E(y)$ from independent considerations, viz. theory and/or experimental data, for the example to be presented is discussed in Ref. 19.

Consider now the base of the body; the only boundary condition to be imposed is:

$$u(0,y,t) = 0 \qquad \text{on} \begin{cases} 0 \leq y \leq H/2 \\ t \geq 0 \end{cases} \quad (7)$$

However, from the $x$-momentum equation, this condition implies $p_x(0,y,t) = 0$. The remaining equations (continuity, $y$-momentum and energy) determine the values of $\rho$, $T$ and $v$ at the base in terms of $\rho_y$, $T_y$ and $u_y$ there, independently of $\rho_x$, $v_x$ and $T_x$. Therefore, the reflection technique suggested by Lax[15] can be employed here to define values of the variables on the column of exterior points $x = -\Delta x$, $0 \leq y \leq H/2$, without loss of generality:

$$\begin{aligned} u(-\Delta x,y,t) &= -u(\Delta x,y,t) \\ v(-\Delta x,y,t) &= v(\Delta x,y,t) \\ \rho(-\Delta x,y,t) &= \rho(\Delta x,y,t) \\ T(-\Delta x,y,t) &= T(\Delta x,y,t) \end{aligned} \qquad \text{on} \begin{cases} 0 \leq y < H/2 \\ t \geq 0 \end{cases} \quad (8)$$

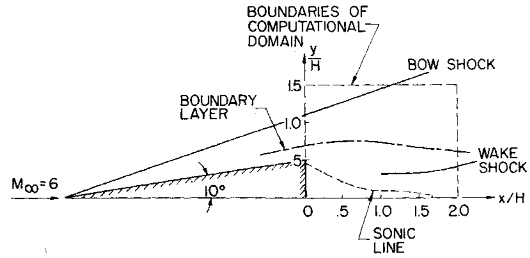


**Fig. 1 Schematic of flowfield about a 10° wedge.**

which corresponds to:

$$\begin{aligned} u_x(0,y,t) &= u(\Delta x,y,t)/\Delta x \\ v_x(0,y,t) &= \rho_x(0,y,t) = T_x(0,y,t) = 0 \end{aligned} \quad (9)$$

Thus, the correct boundary condition at the base, Eq. (7), is satisfied without over-specifying the solution.

The $y$ axis is a plane of symmetry, on which the appropriate boundary condition is:

$$v(x,0,t) = 0 \qquad \text{on} \begin{cases} x \geq 0 \\ t \geq 0 \end{cases} \quad (10)$$

Obviously, within the present context a plane of symmetry cannot be distinguished from an impermeable wall; hence the above discussion of the boundary conditions at the base also pertains to treatment of the boundary conditions on $y = 0$. Again, the reflection technique is used to define the variables at an exterior row of grid points in a manner analogous to that just described above.[19]

Consideration is now given to the two outflow boundaries, $y = y_{\max}$ and $x = x_{\max}$, whose positions are arbitrary in the sense that they do not correspond to any physical boundaries, but rather to practical boundaries on the portion of the flowfield to be determined. Thus, the only requirement that must be satisfied at these boundaries is that they are sufficiently far from the region of interest to have no influence on the solution therein. However, it must be emphasized that use of arbitrary rules for treating the boundary points, such as generalizations of the reflection technique (see, e.g., Burstein[13]), will, in general, result in the generation of a sequence of waves from these boundaries during the transient stages of the solution, which will be trapped within the regions of influence of the boundaries as the asymptotic solution is approached. Moretti[17,18] has pointed out that dramatic consequences can accrue if a region of subsonic outflow occurs at one of these boundaries which allows the waves to travel upstream into the interior of the domain, which Burstein[13] also observed. Such a region of subsonic outflow can be expected near the plane of symmetry in the present problem, although in reality viscous stresses will accelerate the flow to supersonic velocity at some distance from the base.

The crux of the problem at these boundary points is to accurately extrapolate the known solution at time $t$ slightly beyond the computational domain so as to encompass the domain of dependence of the desired solution at time $t + \Delta t$, without over-specifying or constraining the solution and without destabilizing it. Several techniques have been proposed, with limited success.[12–17] The present authors have found the most satisfactory solution to this problem to be an extrapolation procedure using the method of least squares. For example, a set of quadratic equations in $x$ is obtained by curve fitting each of the dependent variables for $y$ = constant = $y_i$ at the grid points $(x_{\max}, y_i)$, $(x_{\max} - \Delta x, y_i)$, $(x_{\max} - 2\Delta x, y_i)$, $(x_{\max} - 3\Delta x, y_i)$, and $(x_{\max} - 4\Delta X, y_i)$ by a standard least squares routine. The quadratic equations are then solved to obtain the values of the variables at the grid point $(x_{\max} + \Delta x, y_i)$. The variables at the grid point $(x_i, y_{\max} + \Delta y)$ are similarly determined by extrapolation in the $y$-direction. The entire procedure is carried out at a fixed time $t$, and the

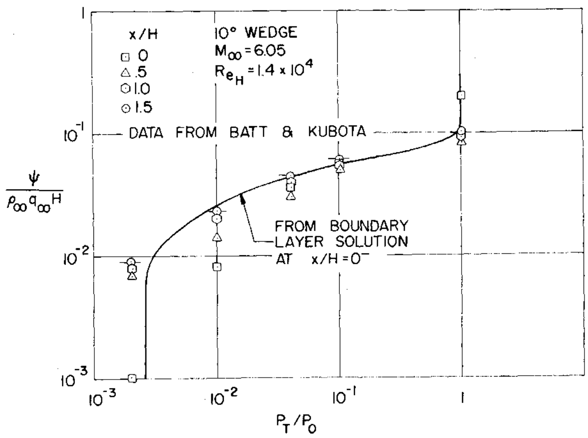


**Fig. 2 Total pressure distribution as a function of mass flow rate in the upstream boundary layer and the wake.**

boundary points can thereby be treated as regular interior points in carrying out the solution for $t + \Delta t$. It is emphasized that the use of the least squares curve fitting routine implies a damping or smoothing of the solution near the boundaries only insorfar as extrapolation of the solution is concerned; the interior points per se are not smoothed. This technique appears to be completely stable so long as the solution at the interior points is reasonably continuous. The only exceptional condition encountered in the present problem occurs where the oblique bow shock crosses the boundary; obviously any attempt to extrapolate from one side of a shock wave to the other is futile. These exceptional points are treated by reverting locally to the reflection rule.

## III. Results for a 10° Wedge at $M_\infty = 6$

### A. Determination of Upstream Boundary Layer

Near-wake calculations have been carried out for conditions corresponding to the experimental results reported by Batt and Kubota[7] which appear to be the most complete and detailed hypersonic, two-dimensional near-wake data presently available. In this connection, it is necessary to describe in detail the properties of the flow entering the region from the upstream boundary layer. However, the experimental results do not resolve the distribution of boundary-layer properties with sufficient accuracy to satisfy the present requirements, viz., determination of the flow variables for Eq. (6); therefore recourse is made to the analysis of Olsson and Meissiter[5] to supplement the experimental data. These considerations are detailed in Ref. 19. The salient conclusions are twofold: first, the flow entering the base region is entirely supersonic; thus disturbances originating in the near wake cannot propagate upstream of the corner. Secondly, the rotational flow entering the base region can be related to the undisturbed upstream boundary layer by an inviscid expansion to the pressure prevailing on each streamline at the corner.

It is of interest to compare the total pressure distribution vs stream function so obtained[19] with the experimental results of Ref. 7. Data points at $X/H = 0^+$, 0.5, 1.0 and 1.5 are shown in Fig. 2, which were determined by cross plotting the streamlines and lines of constant total pressure mapped by Batt and Kubota at $Re_H = 1.4 \times 10^4$ for an adiabatic wall condition. It can be seen that the data agrees quite well with the theoretical boundary-layer distribution at $X/H = 1.5$ but the agreement deteriorates with decreasing distance from the base. Clearly, the correlation should be best at $X/H = 0^+$, and if viscous dissipation were significant it should deteriorate with increasing distance from the base, contrary to the observed behavior. A possible explanation is that the divergence of the flow direction at the corner (it will be shown later that the flow very near the corner turns nearly 90°) caused an erroneously low total-pressure reading due to misalignment of the probe in this region. However, this source of experimental error should become less significant with increasing distance from the base, and be completely negligible at $X/H = 1.5$. Therefore the good agreement between the theoretical curve and the experimental data at this station is regarded corroboration of the inviscid model proposed here. Ohrenberger[6] arrived at similar conclusions by a different approach. However, reliance on the experimental streamline mapping near the corner for initial data apparently leads to a contradiction between inviscid theory and data further downstream of the corner in Ref. 6.

### B. Transient Solution

The flow properties at several points in the near wake were monitored to determine achievement of a steady solution. The base pressure proved to be a reasonably sensitive indicator in this regard. Transient base pressure results are presented in Ref. 19 to demonstrate the attainment of a steady solution and the existence of a limit as $\Phi \to 0$. Execution of the calculations required $2 \times 10^{-3}$ seconds per mesh point per time step on a CDC 6600 computer system. The total flow time to establish a steady solution is approximately $10H/q_\infty$ if the base region is assumed to be at a low pressure initially.[19] It is interesting to note that Zakkay, Sinha, and Medecki[20] measured times of the order of less than $14H/q_\infty$ for the wake of an axisymmetric body at $M_\infty = 4$ to reestablish itself from a perturbed condition. On the other hand they report residence times in the laminar recirculation region of the order of $75H/q_\infty$ for an injected gas, which is clearly a diffusion controlled time scale. Therefore, the present inviscid computation appears to be consistent with the observed transient behavior of the near wake.

### C. Comparison with Experimental Results

The distribution of static pressure along the centerline of the wake is plotted in Fig. 3 for the adiabatic wall condition, with $\delta/H = 0.17$ and 0.10. The mesh size was $\Delta/H = 0.05$‡ in both cases. The corresponding experimental data from Ref. 7 is also shown for comparison. The agreement is quite good for $x/H \leq 1.0$, but at $x/H = 1.5$ the present solutions fall somewhat below the experimental results. The computed base pressure for $\delta/H = 0.10$ is $p_b/p_\infty = 0.2766$ as compared to the experimental value of 0.28 for $\delta/H = 0.09$ and the interpolated value of 0.30 for $\delta/h = 0.10$. The computed value of $p_b/p_\infty = 0.3996$ for $\delta/H = 0.17$ must be regarded as fortuitously close to the experimental value of 0.40 for $\delta/H = 0.16$. Unfortunately, no static pressure data was reported for $x/H < 0.5$; and the value shown at $x/H = 0.75$ is actually a total pressure measurement in the vicinity of the rear stagnation point. The higher pressures observed at $x/H \geq 1.5$

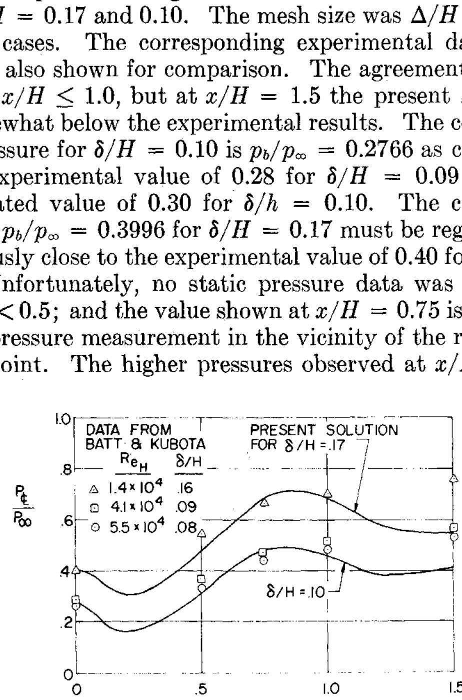


**Fig. 3. Pressure distribution on the wake centerline for an adiabatic wall.**

‡ The resulting artificial viscosity is less than $\frac{1}{10}$ the actual kinematic viscosity in the inviscid flow and $\frac{1}{50}$ the value at the adiabatic wall temperature for $Re_H = 1.4 \times 10^4$.

are probably due to a viscous interaction involving the shear layer developing along the wake centerline beyond the rear stagnation point. The adverse pressure gradient at the rear stagnation point inferred from the data may be regarded as indicative of the importance of viscous effects in the vicinity of this point.

In a related experiment, Hama[8] measured the pressure distribution across the base of a 6° wedge in an $M_\infty = 4.54$ stream. The wake flow remained laminar only at the lowest Reynolds number in his experiments, for which $p_b/p_\infty = 0.35$ was obtained. This data is compared with the present solution for $\delta/H = 0.17$ ($p_b/p_\infty = 0.3996$) in Fig. 4. Although there are obvious quantitative differences in the results near the corner, the curves exhibit certain remarkable similarities, viz., the over-expansion of the flow around the corner ($0.4 < y/H < 0.5$) followed by a recompression and a local maximum around $y/H = 0.35$, another slight expansion, and finally a recompression near $y/H = 0$. The pressure maximum in the present solution at $y/H = 0.36$ corresponds to the location of a stagnation point on the base from which the dividing streamline (i.e., the streamline that encloses the recirculating flow) originates. The overexpansion and recompression are believed to be the cause of the lip shock,[8] although the lip shock per se could not be resolved with certainty in the present calculations.

The normal distributions of static pressure for $\delta/H = 0.17$ are compared with Batt and Kubota's[7] experimental data for $Re_H = 1.4 \times 10^4$ ($\delta/H = 0.16$) at $x/H = 0.05, 0.25, 0.50, 1.0,$ and 1.5 in Fig. 5a and 5b. The agreement is generally quite good, excepting of course in the immediate vicinity of the bow shock. The formation of the wake shock between $0.2 < y/H < 0.4$ is noted in Fig. 5b. As noted above, the presence of a distinct lip shock cannot be discerned from the pressure distributions. The value of the stream function at each mesh point was obtained by numerical integration and the resulting streamline map is compared in Fig. 6 with the experimental map from Ref. 7. Again, good agreement is noted. Also shown is the dividing streamline $\psi = 0$, the sonic line, and the $u = 0$ line. The sonic line location is consistent with the experimental points, but the $u = 0$ line falls much below the experimental data near the base. It can be seen in Ref. 19 that the $u$-velocity profiles become very flat near the base, since $u = 0$ on the base, suggesting that some experimental inaccuracy in defining the $u = 0$ line could easily occur near the base. The variation of the velocity on the wake centerline is plotted in Fig. 7 for the adiabatic wall condition with $\delta/H = 0.17$ and 0.10, and also for a cold wall condition ($T_w/T_0 = 0.19$) with $\delta/H = 0.10$. The recirculation velocity evidently increases with decreasing boundary-layer thickness (or increasing vorticity) for a given total temperature of the recirculating flow; however, the velocity also decreases with decreasing total temperature. The insensitivity of the rear stagnation point location to variations in boundary layer-thickness and wall temperature can also be seen in Fig. 7.

It has been observed experimentally that the base pressure, and indeed the entire pressure distribution along the wake centerline, correlates with the upstream boundary-layer

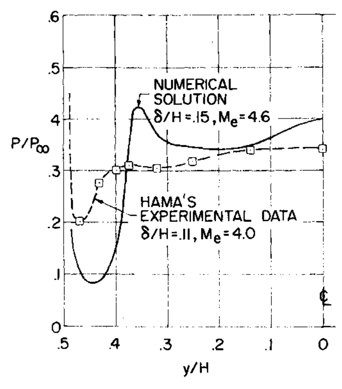


**Fig. 4 Pressure distribution across the base of the wedge.**

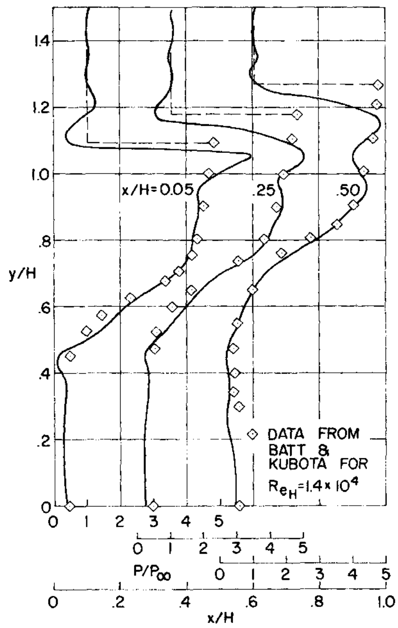


**Fig. 5a Static pressure profiles at several stations in the near wake for $\delta/H = 0.17$ and an adiabatic wall.**

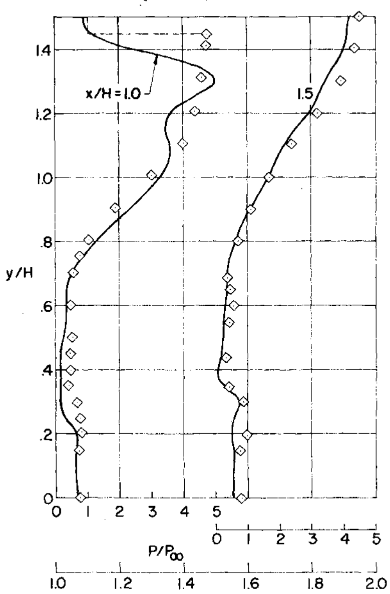


**Fig. 5b Static pressure profiles at several stations in the near wake for $\delta/H = 0.17$ and an adiabatic wall.**

thickness regardless of wall temperature.[7] The computed static pressure distributions along the centerline with $\delta/H = 0.10$ for the adiabatic wall condition and for a cold wall are plotted in Fig. 8 and compared with the related experimental data, and also with cold wall solutions obtained by Reeves and Buss.[21] The present results are qualitatively similar to the experimental data in the sense that there is a small effect of wall temperature relative to the effect of upstream boundary-layer thickness. The quantitative agreement with the data is not nearly so impressive as in the adiabatic wall case, however it correlates more closely with the experimental results than does the Reeves and Buss solution.[21] The present cold wall solution is somewhat less accurate than the comparable adiabatic wall case. The static temperature distribution in the upstream boundary layer is monotonic in the latter case while it possesses a sharp maximum away from the wall in the former case, which is clearly more difficult to approximate numerically. In this sense the use of the total temperature as a dependent variable seems highly desirable. The

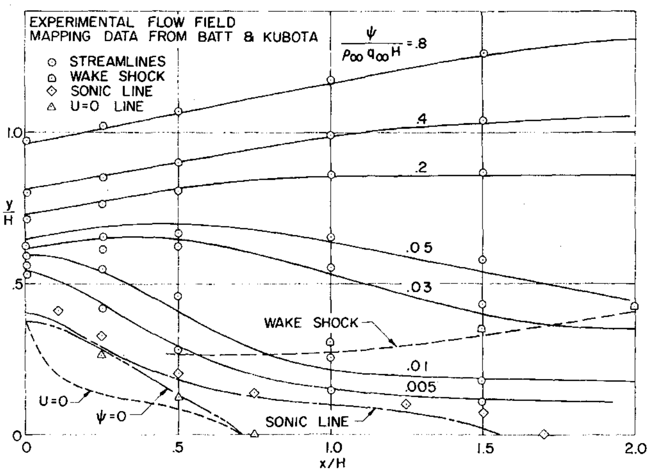


**Fig. 6 Near-wake flowfield map for an adiabatic wall and $\delta/H = 0.17$.**

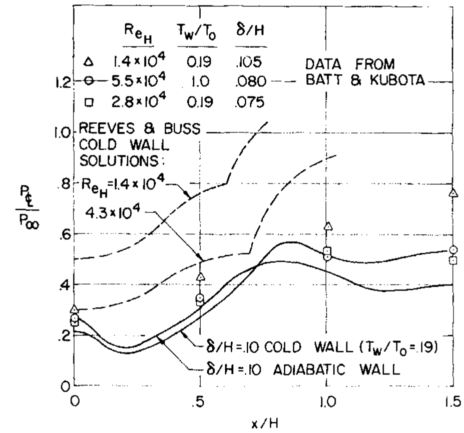


**Fig. 8 Pressure distributions along the wake centerline.**

total temperature distributions at $x/H = 0.75$ and 1.50 from the present method are compared with the experimental data[7] and the Reeves and Buss solution[21] in Fig. 9. The relatively poor description of the total temperature by the present method seems to be its chief deficiency. The static temperature distribution, for example, shows remarkable qualitative agreement with the data reduced by Batt and Kubota,[7] as shown in Fig. 10. The present results are about a factor of two below the experimental values, probably due to the inaccurate representation of the upstream boundary-layer temperature profile, and hence the failure to conserve the actual energy flux entering the near-wake region.

## IV. Conclusions

The present investigation has resulted in the development of a time-dependent finite-difference method for obtaining inviscid solutions of the steady, laminar near wake of a flat-based slender body at supersonic velocities. Consideration of the case of a 10° wedge at $M_\infty = 6$ has afforded the opportunity for detailed comparisons with experimental data.[7] The following conclusions have been drawn from this study:

1) The base pressure, the pressure distributions normal to the axis, and the streamline patterns in the near wake are in general agreement with experimental results[7] for a Reynolds number based on wedge height as low as $1.4 \times 10^4$ and an adiabatic wall condition. However, the pressure distribution along the centerline exhibits a maximum at the rear stagnation point, while the experimental values continuously increase, indicating the importance of viscous effects in the vicinity of this point. The effects of Reynolds number variations on the pressure distributions are accurately predicted in terms of the corresponding variations of the thickness of the upstream boundary layer, and hence the vorticity of the flow entering the region, in accord with the experimentally observed behavior. The recirculation velocity varies inversely with the thickness of the upstream boundary layer (or directly with the vorticity entering the region). (Inclusion of viscous stresses would of course reduce the magnitude of the velocities obtained on the centerline, and the present results may be somewhat unrealistic in this regard.)

2) The static temperature distribution in the upstream boundary layer is more difficult to approximate numerically in the case of a cold wall than in the adiabatic wall case, which results in somewhat less impressive agreement between the present solution and the experimental data in the cold wall case (for the same mesh size). Nevertheless the static temperature profile at $x/H = 0.25$ is qualitatively similar to the data reduced by Batt and Kubota.[7] The static pressure distribution along the wake centerline is in reasonable agreement with the experimental cold wall data.

3) The formation of the wake shock is evident in the computed pressure distributions, and in good agreement with its observed location.[7] The formation of a distinct lip shock cannot be discerned in the present results. However, it is clear from the present solution that the flow at the corner of the base actually turns the corner and starts down the base. However, a stagnation point is formed on the base (e.g., at $y/H \approx 0.35$) which serves to separate the flow from the body. This behavior is in accord with the experiments reported by Hama.[8]

4) The time to establish a steady solution by the present numerical technique, i.e., about $10H/q_\infty$, agrees with the

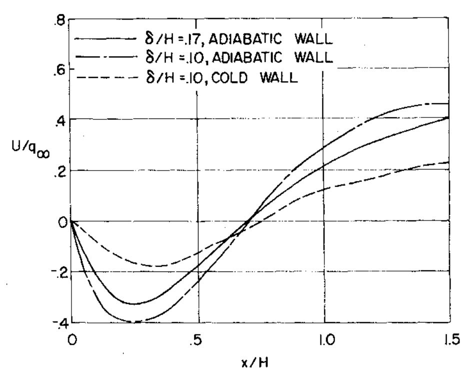


**Fig. 7 Velocity distributions along the wake centerline.**

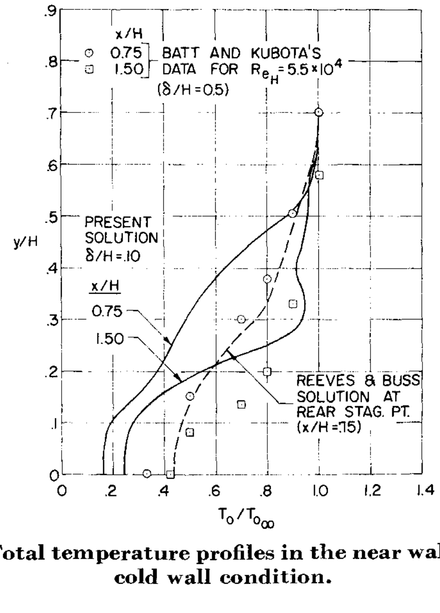


**Fig. 9 Total temperature profiles in the near wake for the cold wall condition.**

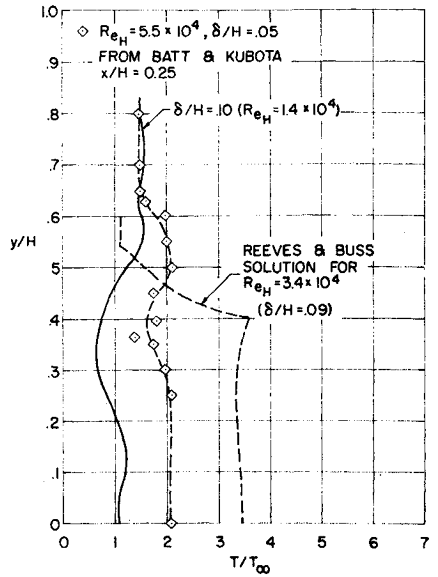


**Fig. 10 Static temperature profile in the near wake for the cold wall condition.**

time for the near wake to re-establish itself from a perturbed condition, measured by Zakkay, Sinha, and Medecki.[20]

Finally, it is pointed out that the present model can be easily extended to incorporate the Navier-Stokes equations rather than the Euler equations as governing conservation laws, with minimal changes in the numerical details[11]; however, accurate description of the free shear layer along the dividing streamline, of the flow near the rear stagnation point and of the boundary layer on the base in the wake will require a grid size of the corresponding scale. Use of an appropriate coordinate transformation or a variable grid size is probably essential in this case.

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